Impossible Differentials Automation: Model Generation and New Techniques

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ID Cryptanalysis automation with Constraint Programming:

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Our goal: improve state of the art of automation of ID Cryptanalysis in the second case. What we do:

- Propose a model for finding Impossible Differential taking into account both direct and indirect transitions.
- Improve the key recovery automation by inserting in the model the hash table method for improved efficiency.

Impossible Differential Cryptanalysis

Idea: exploit differentials with probability 0

Differential Cryptanalysis	ID Cryptanalysis
$\lvert\{P, K \; s.t. \; E(P,K) \oplus E(P \oplus \Delta P, K) = \Delta C\}\rvert$ high	$\lvert\{P, K \; s.t. \; E(P,K) \oplus E(P \oplus \Delta P, K) = \Delta C\}\rvert = 0$

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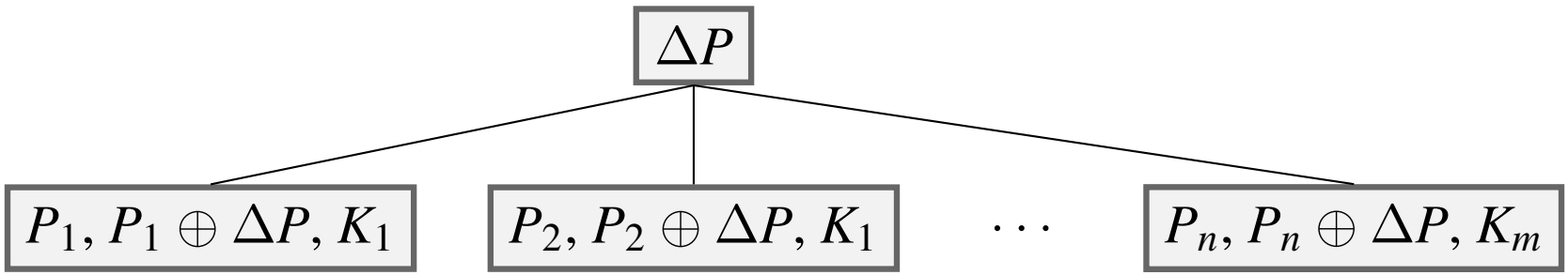

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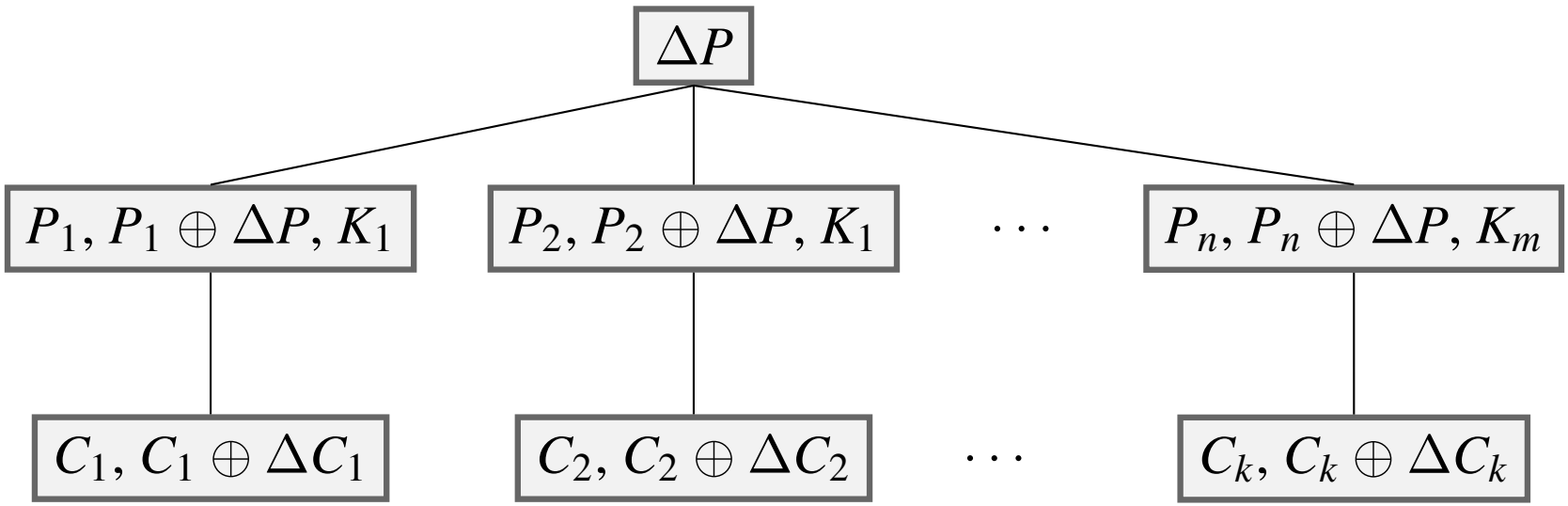

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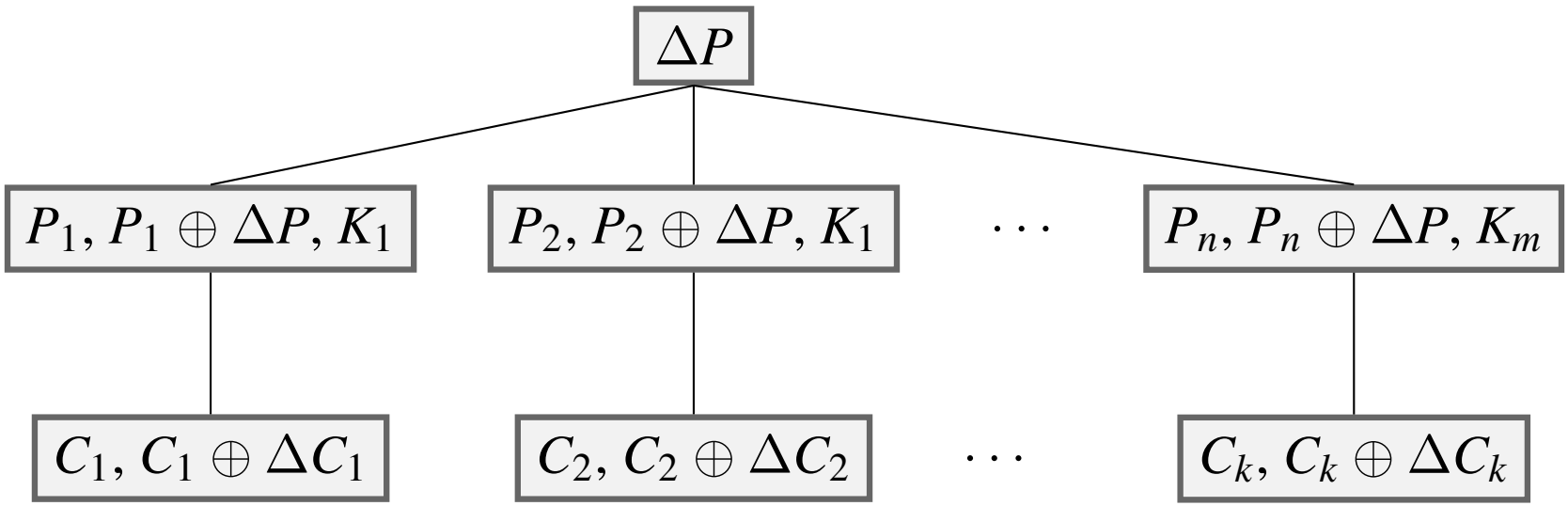

$$\Delta C \notin \{\Delta C_1, \ldots, \Delta C_k\} \implies \Delta P \nrightarrow \Delta C$$

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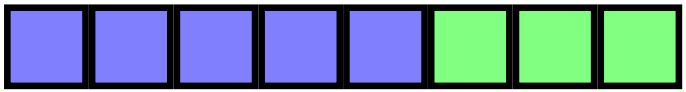
R 1 

R 2

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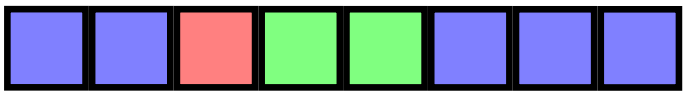
R 5 

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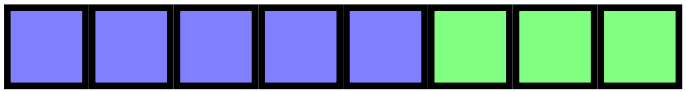

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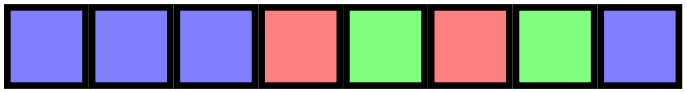

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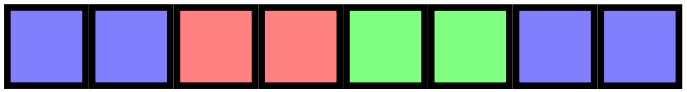

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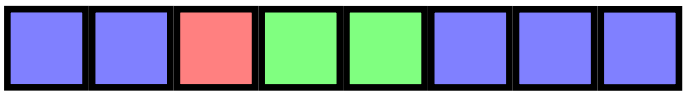

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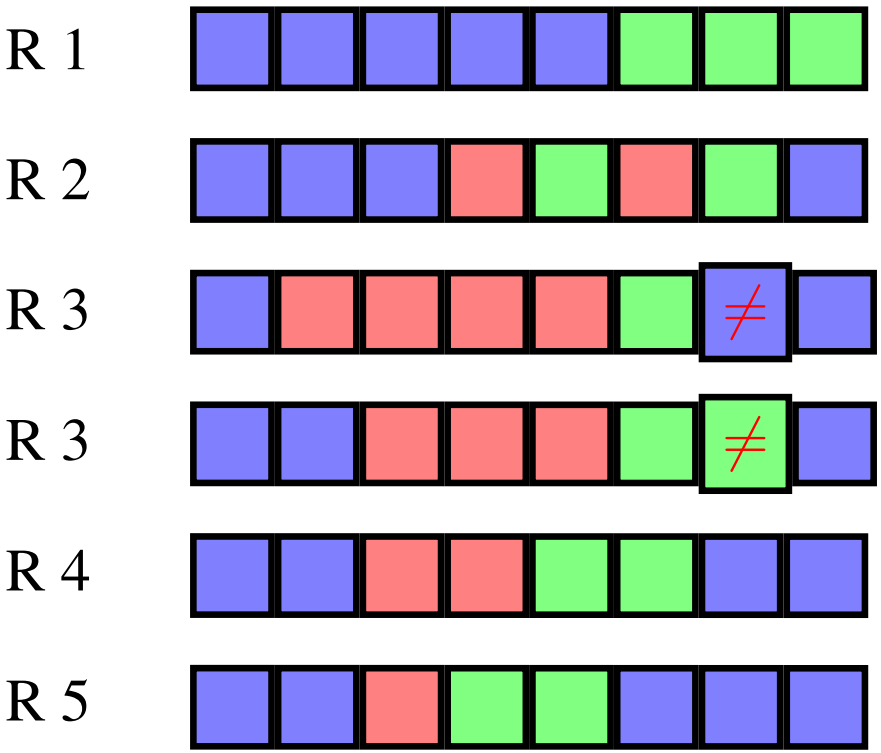

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Bit variables modeling:

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- $\mathit{Unknown} \longrightarrow 2$

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Transition rules for bit operations, e.g.

$$AND(a,b) = \begin{cases} 0 & \text{if } \Delta_a = \Delta_b = 0 \\ 2 & \text{otherwise} \end{cases}$$

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Word variables modeling: differential pattern δ + difference ζ (Sun et al., 2020)

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$$\zeta X \in \begin{cases} \{0\} & \text{if } \delta X = 0 \\ \{1, \ldots, 2^{s-1}\} & \text{if } \delta X = 1 \\ \{-1\} & \text{if } \delta X = 2 \\ \{-2\} & \text{if } \delta X = 3 \end{cases}$$

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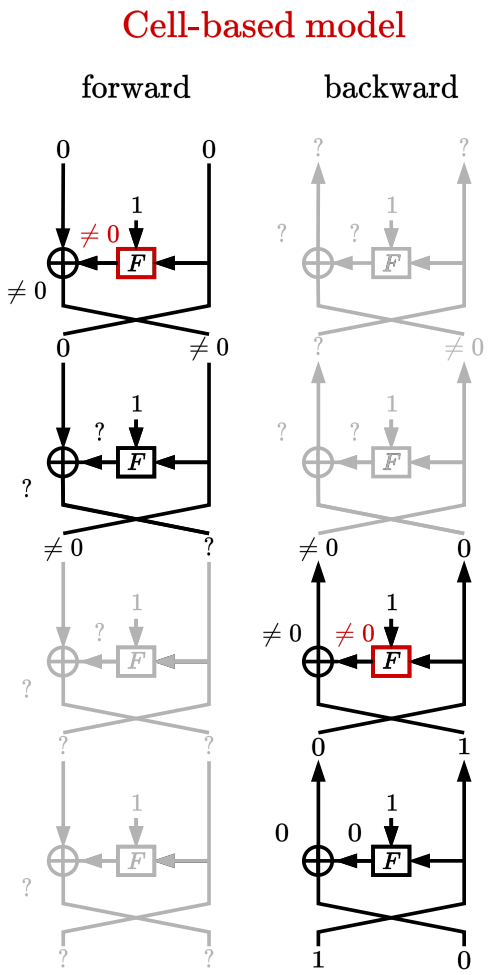
The hybrid model captures both properties in the bit representation:

$$\delta X = \begin{cases} 0 & \text{if } \Delta X_{i,r} = 0 \\ 1 & \text{if } \Delta X_{i,r} = 1 \\ 2 & \text{if } \Delta X_{i,r} \text{ is unknown} \\ id_{s,r'} & \text{if } \Delta X_{i,r} \text{ is produced by S-box } s \text{ of round } r', \\ & \text{evaluated on a nonzero input difference} \end{cases}$$

An example



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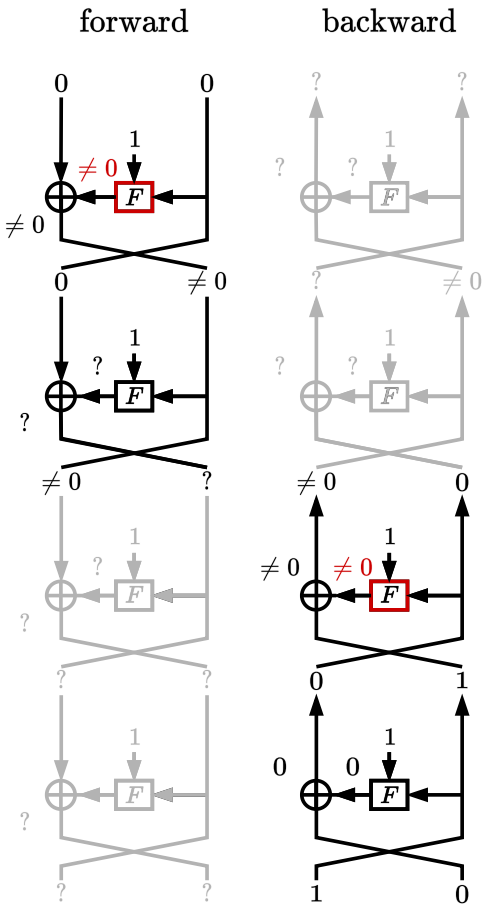
Cell-based model



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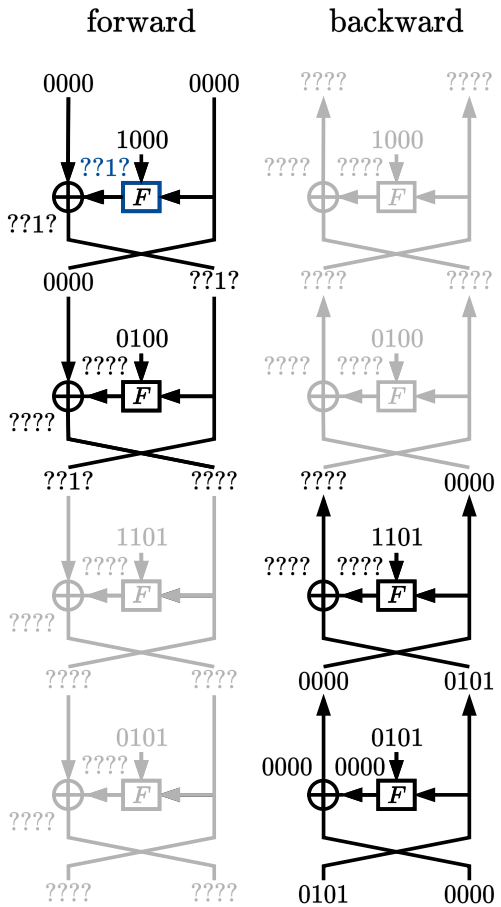

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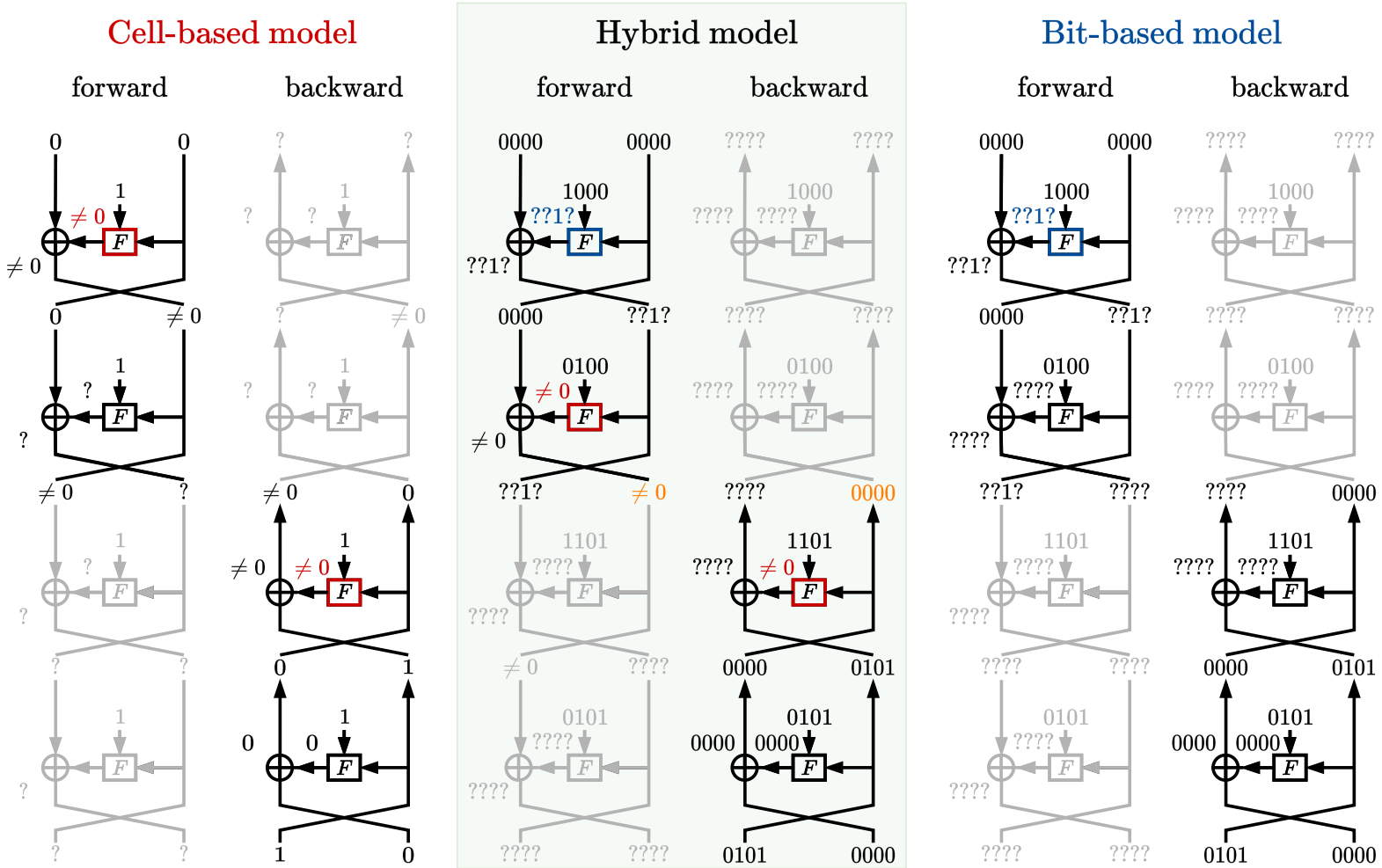
Cell-based model
forward
backward
Hybrid model
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Bit-based model
forward
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LBLOCK

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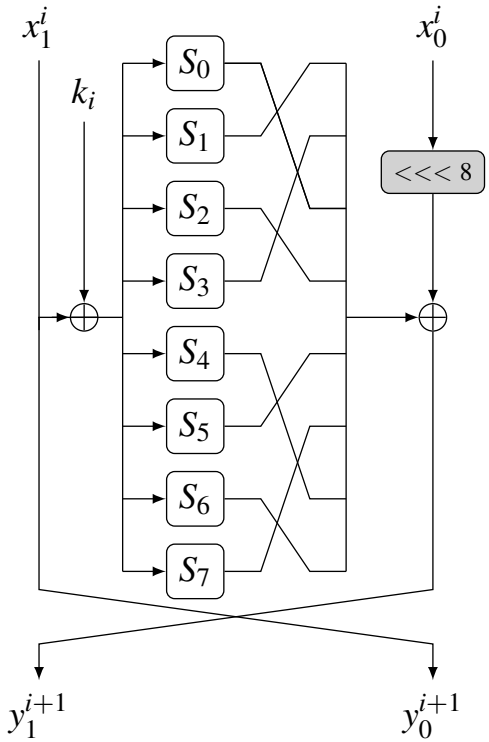

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ID attack (Biham et al., 1998, Knudsen, 1998

Round 1

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Round 5

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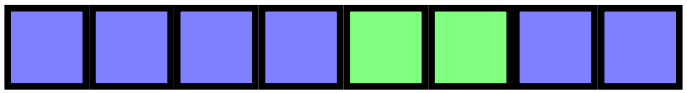

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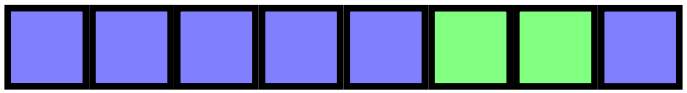

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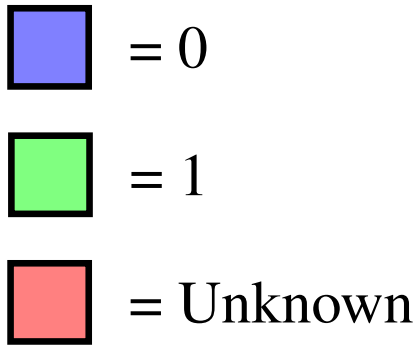

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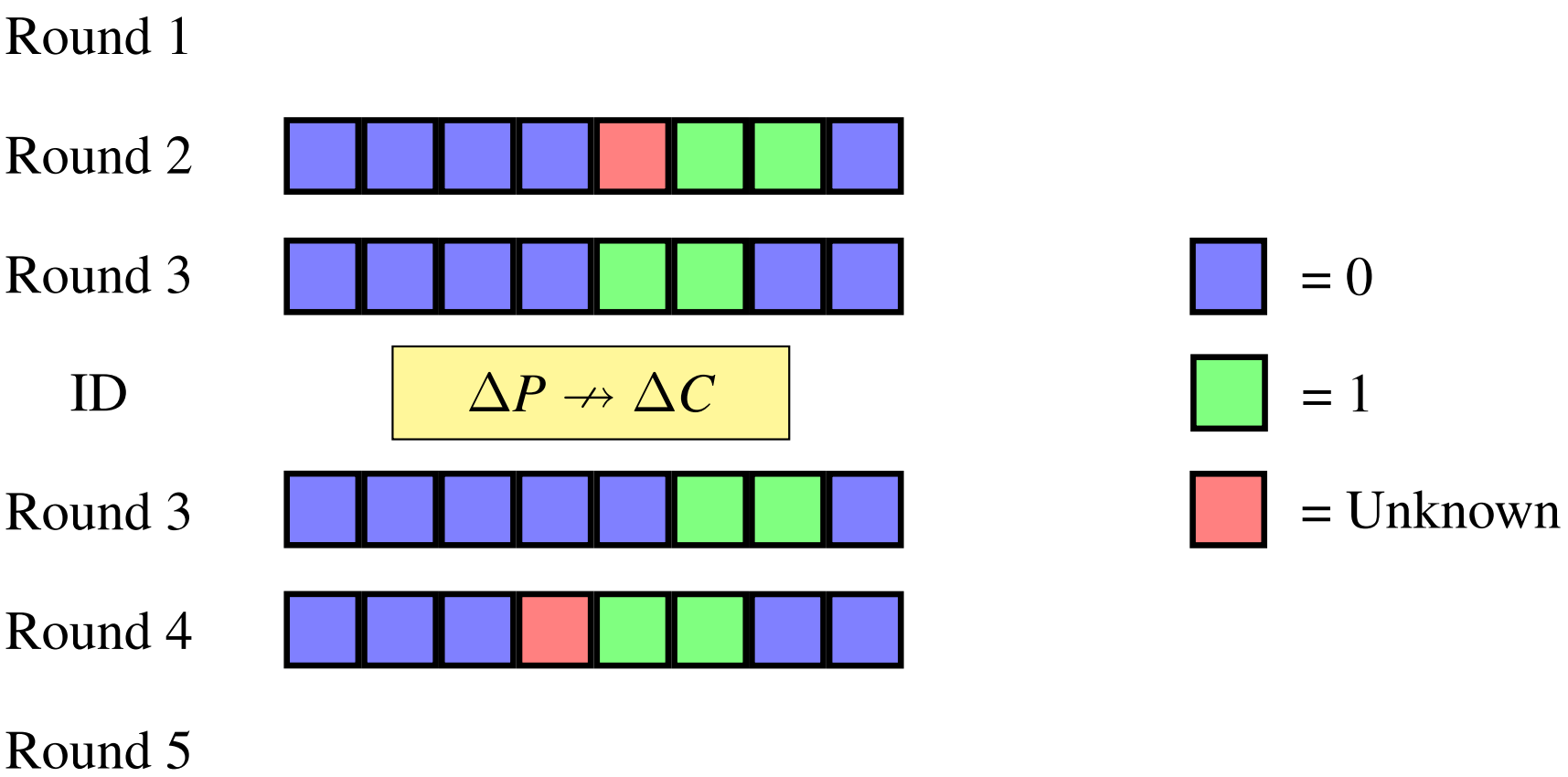

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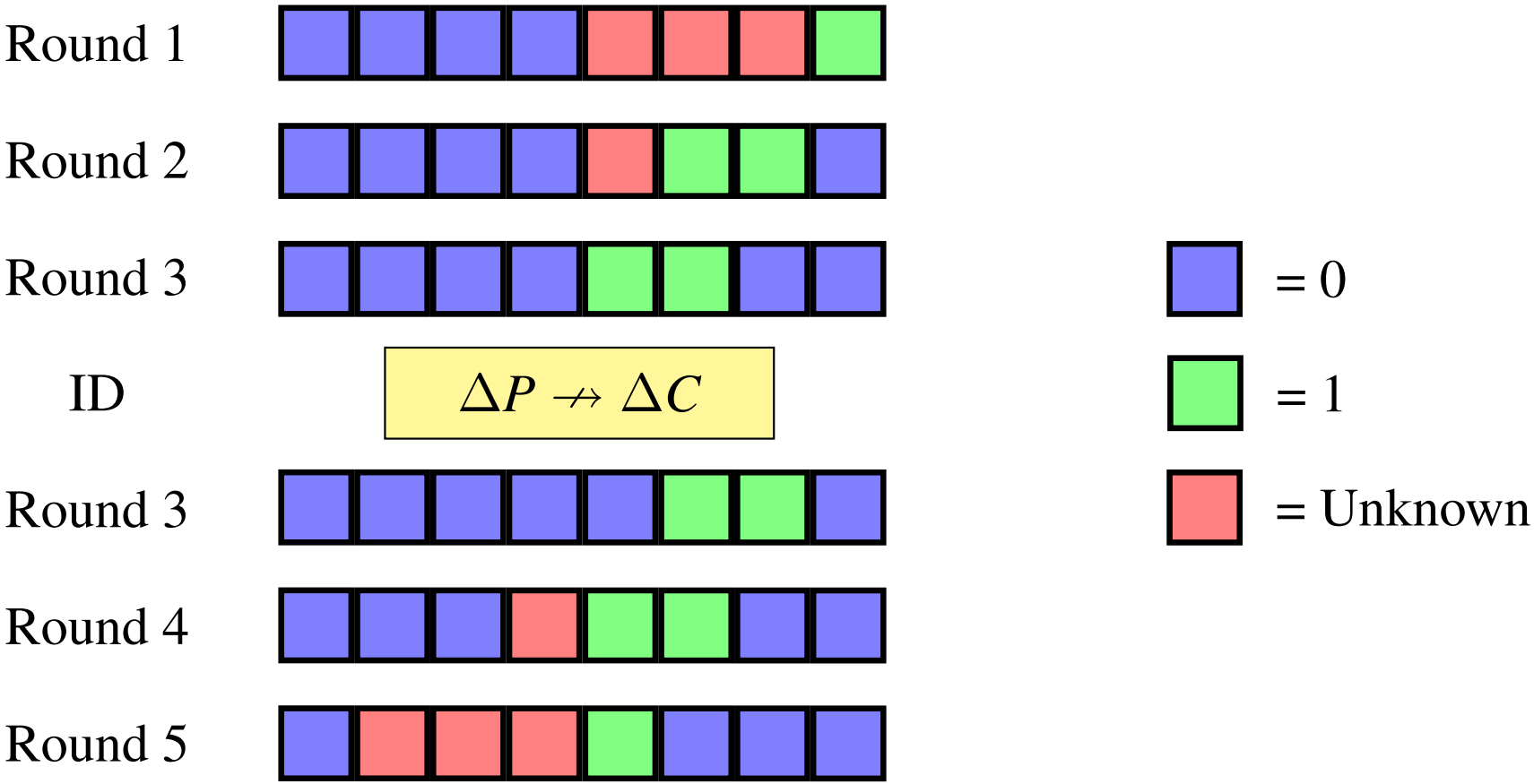

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5. Filter out key guesses which can lead to the impossible differential
6. Exhaustively search the remaining keys

Hash tables for attack efficiency

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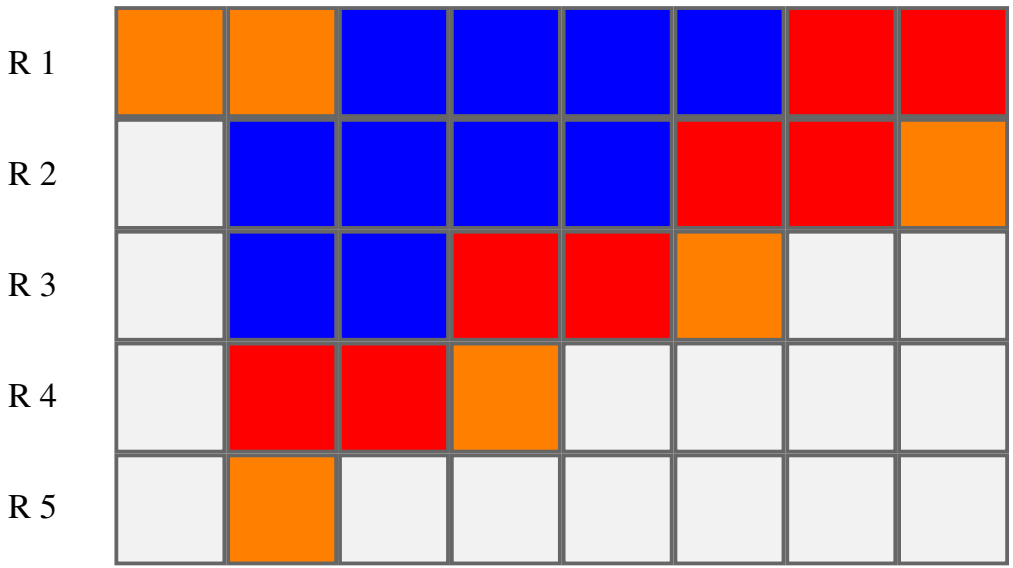

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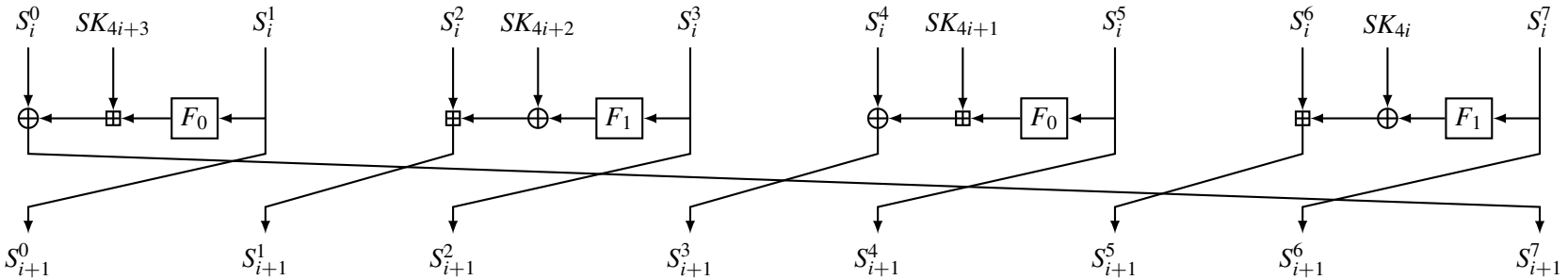

Improved results

Our automatic model for the attack with the hash tables support resulted in an improved 27 rounds attack to the HIGHT block cipher

Scenario	Rounds	Transformations	Time	Data	Memory
Single-key	18 (1-18)	Both	$2^{109.2}$ enc.	$2^{46.8}$ plaintexts	/
Single-key	25 (6-30)	Only final	$2^{126.75}$ enc.	2^{60} plaintexts	/
Single-key	26 (1-26)	Only final	$2^{119.53}$ enc.	2^{61} plaintexts	2^{109} B
Single-key	27 (4-30)	Both	$2^{126.6}$ enc.	2^{58} plaintexts	2^{120} B
Single-key	**27 (4-30)**	**Both**	$2^{124.5}$ **enc.**	2^{60} **plaintexts**	2^{116} **B**

Implementation

Implementation framework: CLAASP library.

Implementation challenges and improvements:

- Cipher inversion
- New hybrid model
- Hash tables inclusion
- Compliance with SAT, MILP and CP formalisms

Summing up

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- Automatic generation of bit-based, cell-based and hybrid models for searching impossible differentials.
- CP modeling of the ID attack including the hash tables efficiency improvement.
- First 18-rounds improbable differential for LBLOCK.
- Most efficient attack on 27-rounds HIGHT.

Thank you!

Any questions?