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SNAKE-EYE RESISTANT AND ROBUST PKE FROM (RING-)LWE WITH SMALL SECRETS

SAC 2026

AMIT DEO BENOÎT LIBERT

Zama

Snake-Eye Resistance & Robustness

δ -Snake-eye resistance (Liu et al., Eurocrypt'25; Mohassel, Asiacrypt'10)

Given pk_1 and pk_2 , finding ct decrypting to the **same** valid plaintext under both keys is hard:

$$\Pr[\text{Dec}(sk_1, ct) = \text{Dec}(sk_2, ct) \neq \perp] \leq \delta$$

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$$\Pr[Dec(sk_1, ct) = Dec(sk_2, ct) \neq \perp] \leq \delta$$

δ -Strong robustness (stronger) (Abdalla-Bellare-Neven, TCC'10)

Hard to find ct decrypting to **any** valid (possibly distinct) plaintexts under both keys:

$$\Pr[Dec(sk_1, ct) \neq \perp \wedge Dec(sk_2, ct) \neq \perp] \leq \delta$$

- Strongly robust $\Rightarrow$ snake-eye resistant
- Black-box transform: snake-eye resistance $\Rightarrow$ strong robustness (Liu et al., Eurocrypt'25)
- **Non-trivial** for key-private PKE (Bellare-Boldyreva-Pointcheval-Desai, Asiacrypt'01)

Outline

Background

- Snake-eye Resistance & Robustness
- Oblivious Message Retrieval
- Hardness Assumptions

Prior work & difficulty of snake-eye resistance with short keys

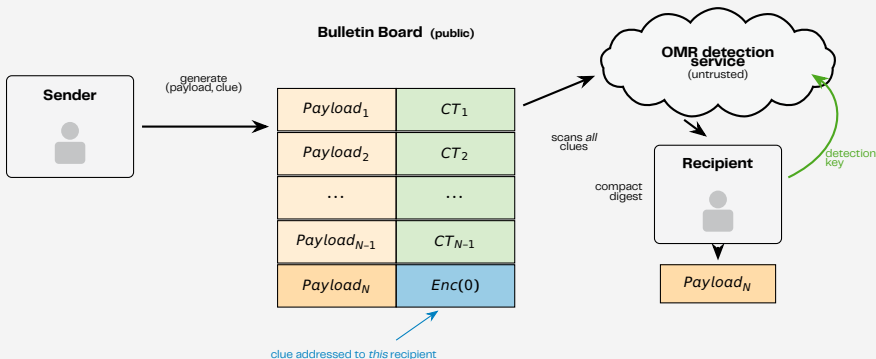
Constructions

- Snake-Eye Resistant PKE from Short-key LWE
- Strongly Robust PKE from Ring-LWE
- Optimized Signaling from RLWE + Binary-Secret LWE

Efficiency Considerations

Motivation: Oblivious Message Retrieval

- Senders post **(payload, clue)** on a public board;
- Recipient delegates detection to an *untrusted* server (Liu-Tromer, Crypto'22)



- **Server learns nothing** about *which* entries belong to the recipient

Motivation: DoS-Resistance in Oblivious Message Retrieval

Bulletin board entries = (payload, **clue**), where a clue is a lightweight PKE ciphertext encrypting 0

- Recipient gives the server an **FHE encryption** of the clue secret key
- Server **homomorphically** decrypts clues

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DoS attack (Liu-Tromer, Crypto'22)

- Malicious sender crafts a clue that is a **false positive for many recipients** at once
- Snake-eye resistance is exactly what rules this out

Two PKE requirements: **snake-eye resistant / robust** and **FHE-friendly decryption**

Hardness Assumptions

The Learning-With-Errors (LWE) problem

Parameters: dimension n , samples $m \geq n$, modulus q .

For secret $\mathbf{s} \leftarrow \chi_s^n$ (small) and error $\mathbf{e} \leftarrow \chi_e^m$ (typically Gaussian), distinguish

$$\left(\begin{array}{c} m \\ \downarrow \\ \text{A} \\ \uparrow \\ n \end{array} \cdot \text{A} \cdot \begin{array}{c} \text{s} \\ \downarrow \end{array} + \begin{array}{c} \text{e} \\ \downarrow \end{array} \right) \text{ VS } \left(\begin{array}{c} \text{A} \\ \downarrow \\ n \end{array} , \begin{array}{c} \text{b} \\ \downarrow \end{array} \right)$$

for uniform $\mathbf{A} \leftarrow U(\mathbb{Z}_q^{m \times n})$ and $\mathbf{b} \leftarrow U(\mathbb{Z}_q^m)$.

- As hard as *worst-case* lattice problems when $\chi_s = \chi_e = D_{\mathbb{Z}, \alpha q}$ with $\alpha q = \Omega(\sqrt{n})$
- Small (binary) secrets keep decryption FHE-friendly – but raise difficulties for snake-eye resistance

Hardness Assumptions

The Ring-Learning-With-Errors (RLWE) problem

Let $R = \mathbb{Z}[X]/(X^N + 1)$, $R_q = R/qR$ and a distribution χ over R

For $s \leftarrow \chi$, distinguish

$$\left\{ (a_i, a_i \cdot s + e_i)_i \mid a_i \leftarrow U(R_q), e_i \leftarrow \chi \right\}_{i=1}^k \quad \text{vs} \quad \left\{ (a_i, b_i) \leftarrow U(R_q^2) \right\}_{i=1}^k$$

$$\left(\begin{array}{c} \boxed{\text{Rot}(a_i)} \\ \boxed{\text{Rot}(a_i)} \end{array} \cdot \begin{array}{c} \boxed{s} \\ \boxed{e_i} \end{array} + \right) \quad \text{vs} \quad \left(\begin{array}{c} \boxed{\text{Rot}(a_i)} \\ \boxed{b_i} \end{array} \right)$$

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$$\left(\begin{array}{c|c} \text{Rot}(a_i) & \text{Rot}(a_i) \cdot s + e_i \end{array} \right) \quad \text{vs} \quad \left(\begin{array}{c|c} \text{Rot}(a_i) & b_i \end{array} \right)$$

The Ring-Short-Integer-Solution (RSIS) problem

Given $\mathbf{a} = (a_1, \dots, a_k) \sim U(R_q^k)$, find a **short, non-zero** $\mathbf{z} = (z_1, \dots, z_k) \in R^k$ s.t. $\langle \mathbf{a}, \mathbf{z} \rangle = 0 \pmod q$

$$\left(\begin{array}{c|c|c} \text{Rot}(a_1) & \text{Rot}(a_2) & \dots & \text{Rot}(a_k) \end{array} \right) \cdot \begin{pmatrix} z_1 \\ z_2 \\ \vdots \\ z_k \end{pmatrix} = \mathbf{0} \in R_q \quad \|z_i\|_\infty \leq \beta$$

Difficulty: Snake-eye Resistance & Short Keys

Prior work (Liu et al., Eurocrypt'25):

- Proved PVW (Crypto'08) and a new **LWE**mongrass scheme snake-eye resistant **under LWE**
- But gave an **attack** on (R)LWE schemes with **small-norm secret keys**

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The attack in one line

Ciphertexts $(\mathbf{a}, b) \in \mathbb{Z}_q^{n+1}$ decrypt via $b - \langle \mathbf{a}, \mathbf{s} \rangle \bmod q$

Output a **small-norm** $(\mathbf{a}, b)$: it decrypts to 0 under **any** sufficiently small key $\mathbf{s}$.

Open: efficient, FHE-friendly, snake-eye resistant PKE **from (R)LWE with small secrets?**

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Idea

Reject $(\mathbf{a}, b)$ if $\|(\mathbf{a}, b)\|_\infty$ is too small

Construction 1: shortLWE with Binary Keys

Variant of (Lindner-Peikert, CT-RSA '11) with a binary key; uses error bound B

- **Keygen:** $\mathbf{A} \leftarrow U(\mathbb{Z}_q^{n \times m})$, $\mathbf{S} \leftarrow U(\{0, 1\}^{n \times \ell})$, $\mathbf{E} \leftarrow \chi_e^{m \times \ell}$

$$\begin{array}{c} m \\ \updownarrow \end{array} \begin{array}{|c|} \hline \mathbf{U}^\top \\ \hline \end{array} = \begin{array}{c} \xrightarrow{n} \\ \begin{array}{|c|} \hline \mathbf{A}^\top \\ \hline \end{array} \end{array} \cdot \begin{array}{c} \xrightarrow{\ell} \\ \begin{array}{|c|} \hline \mathbf{S} \\ \hline \end{array} \end{array} + \begin{array}{|c|} \hline \mathbf{E} \\ \hline \end{array}$$

$$pk = (\mathbf{A}, \mathbf{U})$$

$$sk = \mathbf{S}$$

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- **Encrypt(μ):** $\mathbf{r} \leftarrow U(\{0, 1\}^m)$, $\mathbf{e}_0 \leftarrow \chi_e^n$, $\mathbf{e}_1 \leftarrow \chi_e^\ell$, $\Delta = \lfloor q/p \rfloor$

$$\begin{array}{c} \xrightarrow{m} \\ \boxed{\mathbf{A}} \end{array} \cdot \begin{array}{c} \boxed{\mathbf{r}} \end{array} + \begin{array}{c} \boxed{\mathbf{e}_0} \end{array} = \begin{array}{c} n \\ \updownarrow \\ \boxed{\mathbf{a}} \end{array}$$

$$\begin{array}{c} \ell \\ \updownarrow \\ \boxed{\mathbf{b}} \end{array} = \begin{array}{c} \boxed{\mathbf{U}} \end{array} \cdot \begin{array}{c} \boxed{\mathbf{r}} \end{array} + \begin{array}{c} \boxed{\mathbf{e}_1} \end{array} + \begin{array}{c} \boxed{\Delta \cdot \mu} \end{array}$$

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Snake-eye resistance fix: extra public check in Decrypt

- **Step 1.** Return $\perp$ if $\|\mathbf{a}\|_\infty < 4B$
- **Step 2.** $\mu = \lfloor (\mathbf{b} - \mathbf{S}^T \mathbf{a}) / \Delta \rfloor$; return $\perp$ if $\|\mathbf{b} - \mathbf{S}^T \mathbf{a} - \Delta \cdot \mu\|_\infty > B$; else output μ

Snake-eye Resistance

The correlation LWE (CorLWE) problem (Liu et al., Eurocrypt'25)

For $\mathbf{A}_1, \mathbf{A}_2 \sim U(\mathbb{Z}_q^{n \times m})$, $\mathbf{S}_1, \mathbf{S}_2 \sim \chi_s^{n \times \ell}$, $\mathbf{E}_1, \mathbf{E}_2 \sim \chi_e^{m \times \ell}$, distinguish

$$(\mathbf{S}_1 - \mathbf{S}_2, \{\mathbf{A}_i, \mathbf{A}_i^\top \mathbf{S}_i + \mathbf{E}_i\}_{i=1,2}) \quad \text{from} \quad (\mathbf{S}_1 - \mathbf{S}_2, \{\mathbf{A}_i, \mathbf{U}_i \sim U(\mathbb{Z}_q^{m \times \ell})\}_{i=1,2}),$$

If $\chi_s \sim U(\{0, 1\})$, then **CorLWE** is as hard as **LWE** with $\chi_s \sim U(\{0, 1\})$ in dimension $\approx n/2$.

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If $(\mathbf{a}, \mathbf{b})$ decrypts to μ^* under both $\mathbf{S}_1$ and $\mathbf{S}_2$, then $\|\mathbf{a}\|_\infty \geq 4B$ and

$$\|(\mathbf{S}_1 - \mathbf{S}_2)^\top \mathbf{a} \bmod q\|_\infty \leq 2B$$

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Reduction from CorLWE:

- Build $\{pk_i\}_{i=1}^2$ using $\{(\mathbf{A}_i, \mathbf{U}_i \stackrel{?}{=} \mathbf{A}_i^\top \mathbf{S}_i + \mathbf{E}_i)\}_{i=1}^2$; keep $\tilde{\mathbf{S}} \triangleq \mathbf{S}_1 - \mathbf{S}_2 \in \{-1, 0, 1\}^{m \times \ell}$ internally
- When $\mathcal{A}$ outputs $(\mathbf{a}, \mathbf{b})$, return 1 iff $\tilde{\mathbf{S}}$ satisfies $\|\tilde{\mathbf{S}}^\top \mathbf{a} \bmod q\|_\infty \leq 2B$
- If $\mathbf{U}_1, \mathbf{U}_2 \sim U(\mathbb{Z}_q^{m \times \ell})$, $\tilde{\mathbf{S}}$ is independent of $\mathcal{A}$'s view and $\tilde{\mathbf{S}} \sim \mathcal{D}$ with $\mathcal{D}[0] = 1/2$ and $\mathcal{D}[\pm 1] = 1/4$;

$$\Pr[\|\tilde{\mathbf{S}}^\top \mathbf{a} \bmod q\|_\infty \leq \tfrac{1}{2} \|\mathbf{a}\|_\infty] \leq 2^{-\ell}$$

Construction 2: Randomness-Recovering RLWE PKE

- Cyclotomic ring $R = \mathbb{Z}[X]/(X^n + 1)$, *fully splitting* $R_q = R/(qR)$ with $q \equiv 1 \pmod{2n}$
- Three moduli: q (ciphertext), p (randomness recovery), t (plaintext)
- **Keygen:** Choose $s, e \leftarrow \chi$; set $b = p \cdot (a \cdot s + e)$, re-sample until $b \in R_q^\times$
Output $sk = s$ and $pk = (a, b)$

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$$c_1 = a \cdot r + e_1, \quad c_2 = b \cdot r + e_2 + \mu \cdot \lfloor q/t \rfloor$$

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Robustness mechanism: recover encryption randomness (variant of Hoffmann *et al.*; PKC '23)

- Decrypt rejects if $\|c_1\|_\infty < B$
- Otherwise,
$$c_2 - c_1 \cdot (p \cdot s) \bmod q = (p \cdot \text{noise} + e_2) + \mu \cdot \lfloor q/t \rfloor$$
reveals $\mu \in R_t$, and then $(e_2, r, e_1) \in R^3$ (rejecting if $\|(e_2, r, e_1)\|_\infty > B$)
- A non-rejected c_1 certifies small $(r, e_1) \in R^2$ with $c_1 = a \cdot r + e_1$

Strong Robustness under Ring-SIS

A collision $(r, e_1) \in R^2$, $(r', e'_1) \in R^2$ means one c_1 opens under both keys:

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Reduction obtains a **short** $\mathbf{v} = (r, -r', e_1 - e'_1) \in R^3$ with

$$\left(\begin{array}{|c|} \hline \text{Rot}(a) \\ \hline \end{array} \begin{array}{|c|} \hline \text{Rot}(a') \\ \hline \end{array} \begin{array}{|c|} \hline \mathbf{I}_n \\ \hline \end{array} \right) \cdot \underbrace{\left(\begin{array}{|c|} \hline r \\ \hline -r' \\ \hline e_1 - e'_1 \\ \hline \end{array} \right)}_{\triangleq \mathbf{v}} = \mathbf{0} \in R_q$$

- $\mathbf{v} \neq \mathbf{0}$: if $\|c_1\|_\infty < B$, ciphertext is rejected ($\mathbf{v} = \mathbf{0} \Rightarrow r = r' = 0$ and $c_1 = e_1 = e'_1$)
- Solves an $RSIS_{n,k,q,\beta}$ instance $(a_1, \dots, a_k) \in R_q^k$: set $a = a_1 a_i^{-1}$, $a' = a_2 a_i^{-1}$ with $a_i \in R_q^\times$

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Result

Strong robustness under $RSIS_{n,k,q,\beta}$, with $k = O(\lambda/\log(q/n))$ and $q > n$

First strongly robust PKE from RLWE **in the standard model**

Problem: Expensive Homomorphic Detection

Homomorphically detecting encryptions of 0 is expensive

- Computing $e_2 = \zeta \bmod p$ is **non-linear** — needs a homomorphic reduction mod p
- Via interpolation (Iliashenko-Nègre-Zucca, WHAC '21):

$\approx \sqrt{q}$ homomorphic mults, $\lceil \log(q-1) \rceil + 1$ levels

- At depth > 30 : $N \geq 2^{16}$, so that $q \geq 786433$ and $\sqrt{q} - 3 > 886$ mults *per* mod- p reduction

Bottleneck

For $n = 1024$: $\approx$ **one million** homomorphic multiplications just for the mod- p step (prohibitive)

Construction 3: A KEM-DEM Twist

Idea: replace randomness recovery with encapsulation of a **binary** secret

- **Encapsulate** a binary $\mathbf{r} \in \{0, 1\}^d$ with a truncated LPR ciphertext (Lyubashevsky-Peikert-Regev, Eurocrypt'10)
- Commit to $\mathbf{r}$ via $\mathbf{c}_3 = \mathbf{A}^\top \mathbf{r} + \mathbf{e}_3$
- Binary $\mathbf{r} \Rightarrow$ homomorphic decoding uses **two batched range checks** (no homomorphic rounding / mod- p)

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Why LWE and RLWE?

- $\mathbf{c}_3$ is a **statistically binding commitment** to $\mathbf{r}$ (**statistical** snake-eye resistance)
- LWE dimension d is flexible: can be $< n$ and need not be a power of 2 (shorter ciphertext)
- Deriving $\mathbf{A}$ from a random oracle $\Rightarrow$ public key is one ring element $\mathbf{b} \in R_q$

Concrete Performance of Construction 3

Removing the mod- p reduction cuts the dominant cost of Construction 2

Example parameters (128-bit, $\epsilon_n \approx 2^{-30}$)	
RLWE dim $n = 1024$	LWE dim $d = 680$, $m = 124$
Hamming weight $h = 80$	$B_e = B' = 45$
BFV ring $N = 2^{17}$	plaintext mod $q = 786433$
mult. depth $= 61$	clue key ≈ 2.44 kB

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- **Good:** public/clue keys $\approx 2\times$ smaller than LWE solutions
- **Still:** homomorphic detection slower than **LWEmongrass** (Liu et al., Eurocrypt'25) due to $d = 680$ affine transforms and two sequential batch range checks

Takeaways & Open Problems

What we showed

- A **simple public norm check** makes short-key LWE PKE snake-eye resistant (binary *and* binomial secrets)
- First **RLWE** robust PKE *without* KEM–DEM or random oracles: randomness recovery + Ring–SIS assumption (or statistically, in partially splitting rings)
- Optimized RLWE+LWE with $2\times$ smaller keys and **statistical** snake-eye resistance

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Open

- Close the gap to LWE-based OMR performance (fewer affine transforms / range checks)
- Ring analogue of the Construction-1 argument (correlated *ring*-LWE)

First step towards DoS-resistant OMR under structured lattice assumptions

Contact and Links

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