Integral Resistance and Degree Bounds for Complex Linear Layers

Application to PRINCE and Lower-Latency Alternatives

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- Difficult to give guarantees against integral distinguisher
  - Integral resistance provides strong arguments against integral distinguisher
- Hard to show integral resistance for ciphers with large linear layers
  - Zeng and Tian [ZT25] introduce minimal transition method
- We generalize this idea and provide some theoretical foundations

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- Given an input set, sum over the corresponding outputs

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$$E_k^{(i)}(\boldsymbol{x}) = \sum_{\boldsymbol{u} \in \mathbb{F}_2^n} a_{\boldsymbol{u}}^{(i)}(\boldsymbol{k}) \cdot \boldsymbol{x}^{\boldsymbol{u}} = \sum_{\boldsymbol{u} \in \mathbb{F}_2^n, \boldsymbol{v} \in \mathbb{F}_2^m} a_{\boldsymbol{u},\boldsymbol{v}}^{(i)} \cdot \boldsymbol{k}^{\boldsymbol{v}} \boldsymbol{x}^{\boldsymbol{u}}$$

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- Example: Xor

$$E(\boldsymbol{x}) = 0 \cdot x^{00} + \mathbf{1} \cdot x^{01} + \mathbf{1} \cdot x^{10} + 0 \cdot x^{11} = x_1 + x_2$$

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0	1	1
1	0	1
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1	0	1
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- When $a_{\boldsymbol{u}} = 0$, saturating the input space $\boldsymbol{u}$ leads to an integral property
- Degree d of function
  - $d = \max_{\boldsymbol{u}} hw(\boldsymbol{u})$ where $a_{\boldsymbol{u}} = 1$

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- With the $n-1$ degree input monomials ($u = \bar{\boldsymbol{e}}_1 = 011\ldots1$) we construct

$$\mathcal{I}(E) = \begin{pmatrix} a^{(1)}_{\bar{\boldsymbol{e}}_1,\boldsymbol{v}_1} & a^{(2)}_{\bar{\boldsymbol{e}}_1,\boldsymbol{v}_1} & \cdots & a^{(n)}_{\bar{\boldsymbol{e}}_1,\boldsymbol{v}_1} & a^{(1)}_{\bar{\boldsymbol{e}}_2,\boldsymbol{v}_1} & \cdots & a^{(j)}_{\bar{\boldsymbol{e}}_i,\boldsymbol{v}_1} & \cdots & a^{(n)}_{\bar{\boldsymbol{e}}_n,\boldsymbol{v}_1} \\ a^{(1)}_{\bar{\boldsymbol{e}}_1,\boldsymbol{v}_2} & a^{(2)}_{\bar{\boldsymbol{e}}_1,\boldsymbol{v}_2} & \cdots & a^{(n)}_{\bar{\boldsymbol{e}}_1,\boldsymbol{v}_2} & a^{(1)}_{\bar{\boldsymbol{e}}_2,\boldsymbol{v}_2} & \cdots & a^{(j)}_{\bar{\boldsymbol{e}}_i,\boldsymbol{v}_2} & \cdots & a^{(n)}_{\bar{\boldsymbol{e}}_n,\boldsymbol{v}_2} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ a^{(1)}_{\bar{\boldsymbol{e}}_1,\boldsymbol{v}_s} & a^{(2)}_{\bar{\boldsymbol{e}}_1,\boldsymbol{v}_s} & \cdots & a^{(n)}_{\bar{\boldsymbol{e}}_1,\boldsymbol{v}_s} & a^{(1)}_{\bar{\boldsymbol{e}}_2,\boldsymbol{v}_s} & \cdots & a^{(j)}_{\bar{\boldsymbol{e}}_i,\boldsymbol{v}_s} & \cdots & a^{(n)}_{\bar{\boldsymbol{e}}_n,\boldsymbol{v}_s} \end{pmatrix}$$

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  - With key whitening statement holds for any input set

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0	×	·
1	·	×
2	·	×
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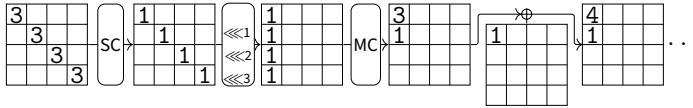

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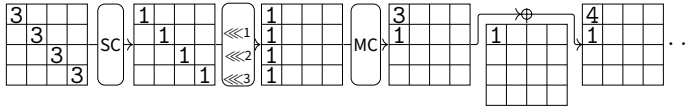

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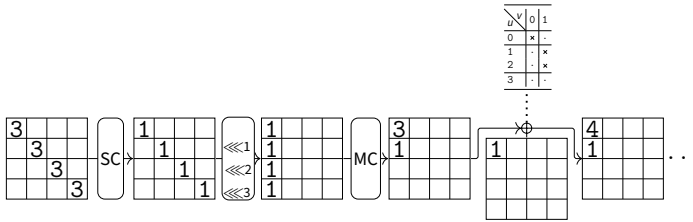

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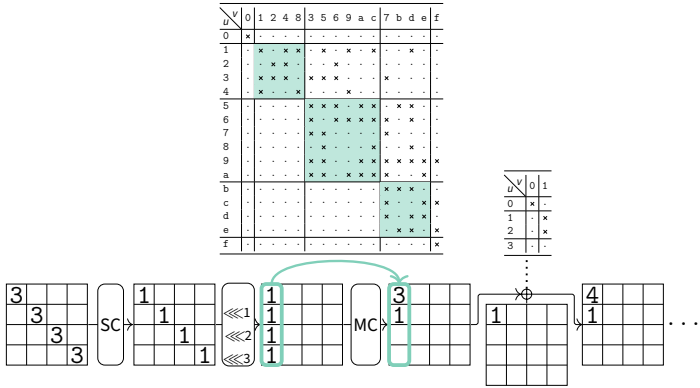

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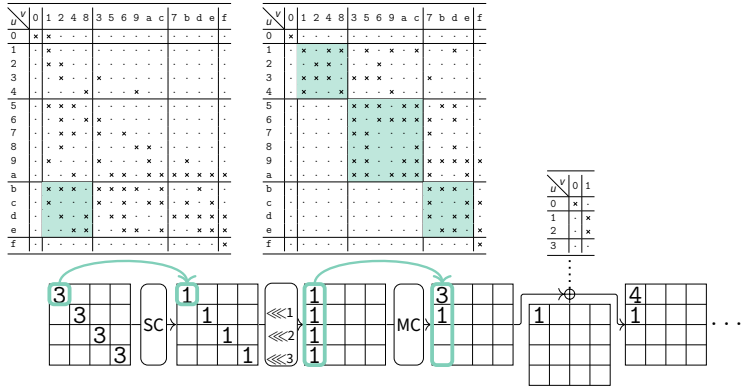

S-Box Requirements

Complete S-Box [ZT25]

If the monomial $\boldsymbol{x}^{\boldsymbol{u}}$ appears in the ANF of some output bit y_i for every vector $\boldsymbol{u}$ with $hw(\boldsymbol{u}) = n - 1$, then S is called a complete S-box.

u \ v	0	1	2	4	8	3	5	6	9	a	c	7	b	d	e	f
0	×	×	·	·	·	·	·	·	·	·	·	·	·	·	·	·
1	·	×	·	·	·	·	·	·	·	·	·	·	·	·	·	·
2	·	×	×	·	·	·	·	·	·	·	·	·	·	·	·	·
3	·	·	×	·	·	×	·	·	·	·	·	·	·	·	·	·
4	·	·	·	·	×	·	·	·	×	·	·	·	·	·	·	·
5	·	×	×	×	·	·	·	·	·	·	·	·	·	·	·	·
6	·	·	×	·	×	×	·	·	·	·	·	·	·	·	·	·
7	·	·	×	×	·	×	·	×	·	·	·	·	·	·	·	·
8	·	·	×	·	·	·	·	·	×	×	·	·	·	·	·	·
9	·	×	·	·	·	×	·	·	·	×	·	·	×	·	·	·
a	·	·	·	×	·	·	×	×	·	×	×	×	×	×	×	×
b	·	×	×	×	·	×	×	×	×	·	×	·	·	×	·	·
c	·	×	·	·	·	×	·	×	·	×	×	·	×	·	×	·
d	·	·	×	·	×	·	×	×	·	·	·	×	×	×	×	×
e	·	·	·	×	×	·	·	×	×	·	×	·	·	×	×	×
f	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	×

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- This is not a sufficient criterion

u \ v	0	1	2	4	8	3	5	6	9	a	c	7	b	d	e	f
0	×	×	·	·	·	·	·	·	·	·	·	·	·	·	·	·
1	·	×	·	·	·	·	·	·	·	·	·	·	·	·	·	·
2	·	×	×	·	·	·	·	·	·	·	·	·	·	·	·	·
3	·	·	×	·	·	×	·	·	·	·	·	·	·	·	·	·
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6	·	·	×	·	×	×	·	·	·	·	·	·	·	·	·	·
7	·	·	×	×	·	×	·	×	·	·	·	·	·	·	·	·
8	·	·	×	·	·	·	·	·	×	×	·	·	·	·	·	·
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a	·	·	·	×	·	·	×	×	·	×	×	×	×	×	×	×
b	·	×	×	×	·	×	×	×	×	·	×	·	·	×	·	·
c	·	×	·	·	·	×	·	×	·	×	×	·	×	·	×	·
d	·	·	×	·	×	·	×	×	·	·	·	×	×	×	×	×
e	·	·	·	×	×	·	·	×	×	·	×	·	·	×	×	×
f	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	×

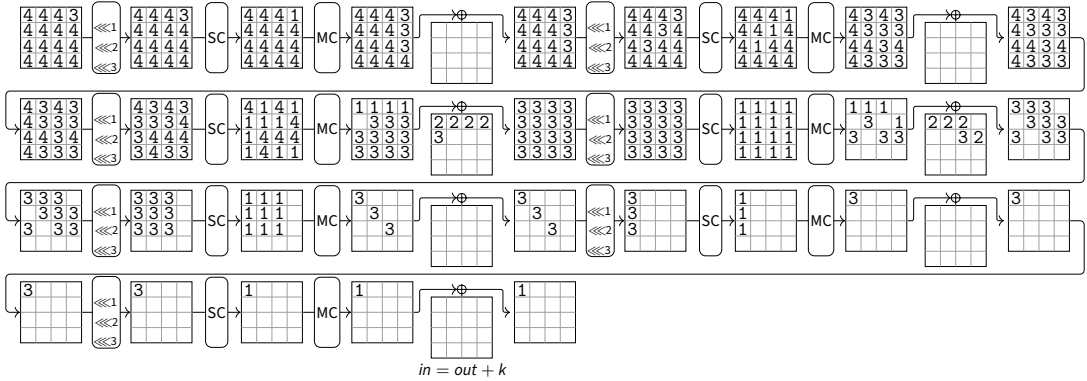
S-Box Requirements

Max-degree full-rank S-Box

A S-Box is max-degree full-rank iff the submatrix of the MPT encompassing all transitions $\boldsymbol{x}^{\boldsymbol{u}}$ to $\boldsymbol{y}^{\boldsymbol{w}}$, with $hw(\boldsymbol{u}) = n - 1$ and $hw(\boldsymbol{w}) = 1$, is full rank.

u \ v	0	1	2	4	8	3	5	6	9	a	c	7	b	d	e	f
0	×	×	·	·	·	·	·	·	·	·	·	·	·	·	·	·
1	·	×	·	·	·	·	·	·	·	·	·	·	·	·	·	·
2	·	×	×	·	·	·	·	·	·	·	·	·	·	·	·	·
3	·	·	×	·	·	×	·	·	·	·	·	·	·	·	·	·
4	·	·	·	·	×	·	·	·	×	·	·	·	·	·	·	·
5	·	×	×	×	·	·	·	·	·	·	·	·	·	·	·	·
6	·	·	×	·	×	×	·	·	·	·	·	·	·	·	·	·
7	·	·	×	×	·	×	·	×	·	·	·	·	·	·	·	·
8	·	·	×	·	·	·	·	·	×	×	·	·	·	·	·	·
9	·	×	·	·	·	×	·	·	·	×	·	·	×	·	·	·
a	·	·	·	×	·	·	×	×	·	×	×	×	×	×	×	×
b	·	×	×	×	·	×	×	×	×	·	×	·	·	×	·	·
c	·	×	·	·	·	×	·	×	·	×	×	·	×	·	×	·
d	·	·	×	·	×	·	×	×	·	·	·	×	×	×	×	×
e	·	·	·	×	×	·	·	×	×	·	×	·	·	×	×	×
f	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	×

Degree Abstraction Model



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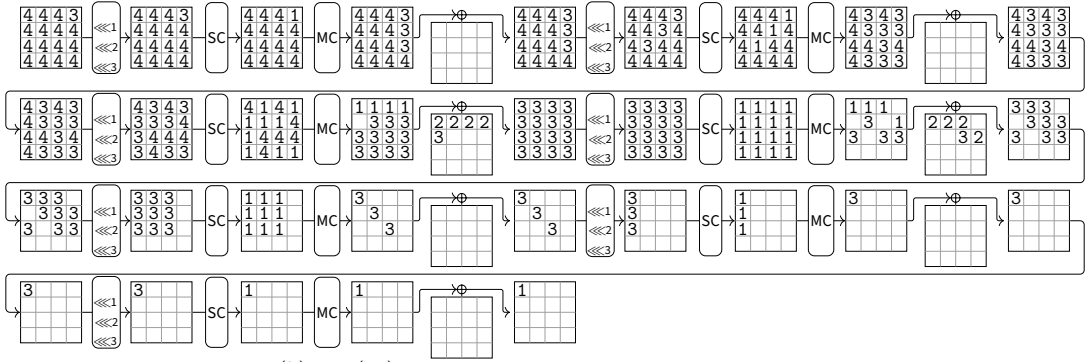
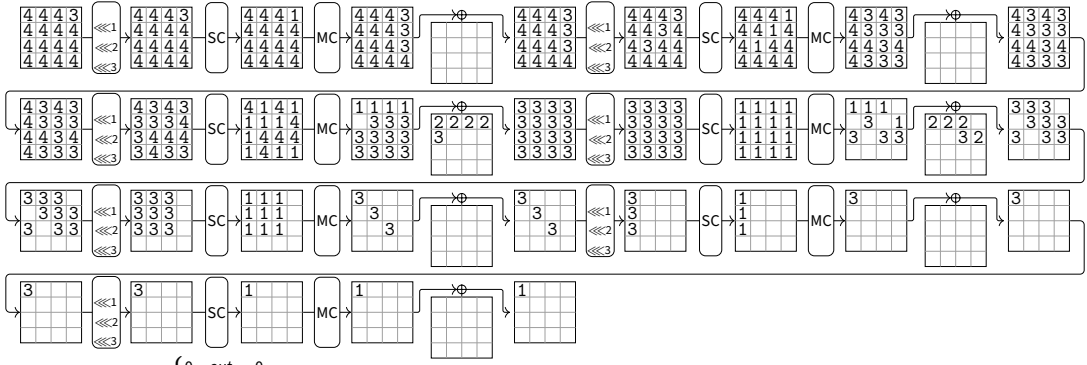


$sum(in) \leq sum(out)$

Degree Abstraction Model



$$in = \begin{cases} 0 & out = 0 \\ 4 & out = 4 \\ 3 & out = 1 \end{cases}$$

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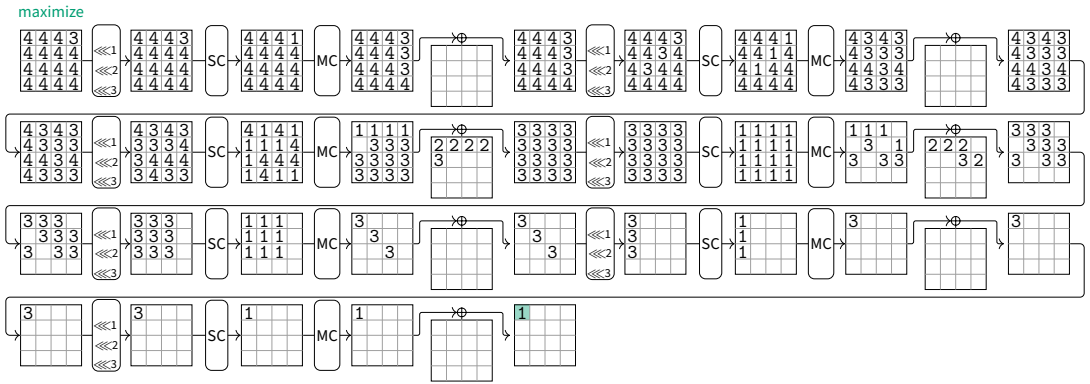

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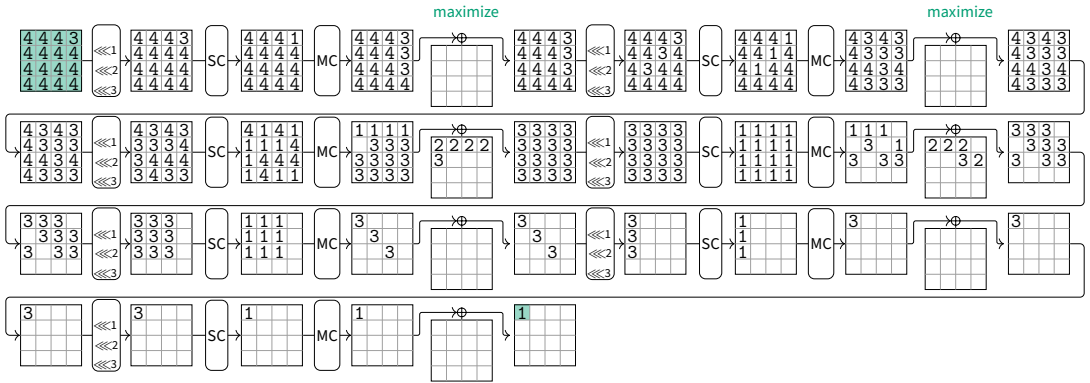

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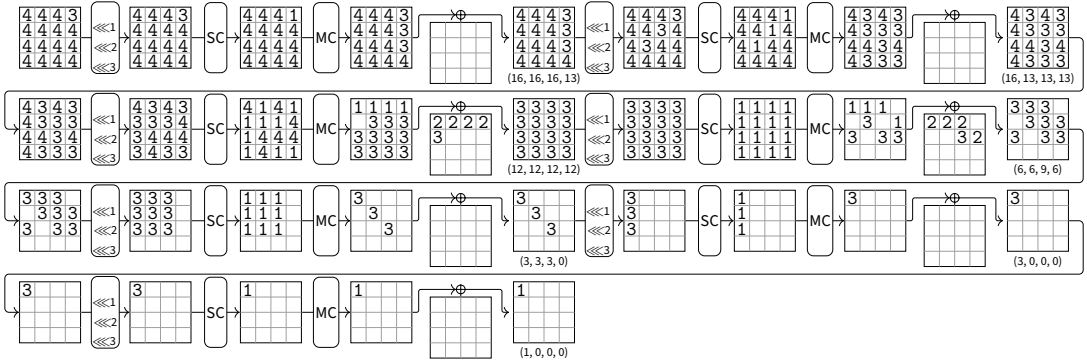

Degree Abstraction Results

Cipher	Model	1	2	3	4	5	6	7
PRINCE like	Cell-Based	3	9	27	49	57	61	63
PRINCE with 64-bit MC	Cell-Based	3	9	27	51	59	62	**63**
Rijndael-256	Bound [BC13]	7	28	113	235	250	254	255
	Cell-Based	7	28	**112**	**232**	**249**	254	255

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- General property of PRINCE like ciphers without max-degree full-rank S-Boxes

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PRINCE with 64-bit MC	Cell-Based	3	9	27	51	59	62	**63**
Rijndael-256	Bound [BC13]	7	28	113	235	250	254	255
	Cell-Based	7	28	**112**	**232**	**249**	254	255

- General property of PRINCE like ciphers without max-degree full-rank S-Boxes
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Degree Abstraction Results

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- General property of PRINCE like ciphers without max-degree full-rank S-Boxes
  - Force degree 3 to 2 transition in first round
  - 7 round model becomes unsatisfiable
    - Always 7 round distinguisher for ciphers like LED, uKNIT

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   - At input we get all $n - 1$ degree monomials contained in the ANF of output

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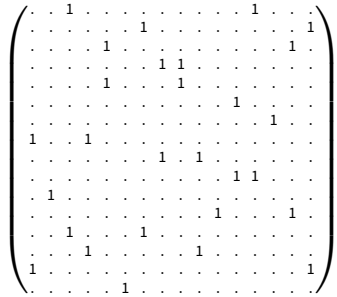

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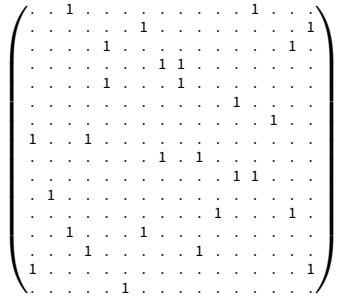

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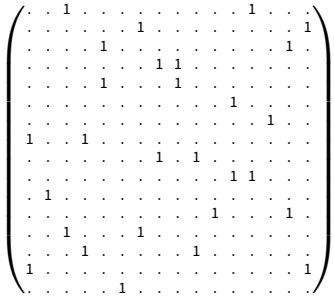
  - Allows us to verify correctness of approach with trail counting



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</> `https://extgit.isec.tugraz.at/castle/tool/intres`

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